

### 4.13 Properties of Logarithms

**Memorize the following:**

**Definition of Logarithm:**  $y = \log_a x \Leftrightarrow a^y = x$

**Properties of Logarithms:** For any positive numbers  $M$ ,  $N$ , and  $a$ , where  $a \neq 1$  and  $r$  is any real number:

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$a^{\log_a M} = M$$

$$\log_a a^r = r$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_a M^r = r \log_a M$$

$$\log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a M = \log_a N \Leftrightarrow M = N$$

**Change of Base Formula:**  $\log_a M = \frac{\log_b M}{\log_b a}$

$$\log_a M = \frac{\log M}{\log a}$$

$$\log_a M = \frac{\ln M}{\ln a}$$

**Examples:** Find the exact value of each expression. (Do not use a calculator).

a)  $\log_{0.6} 0.6^{-3.2}$

b)  $5^{\log_5 3}$

c)  $\log_7 7^{-1}$

d)  $e^{\ln 12}$

**Examples:** Use the change of base formula to evaluate each logarithm.

a)  $\log_6 9$

b)  $\log_{\sqrt{2}} 7$

c)  $\log_{\pi} \sqrt{3}$

d)  $\log_3 5$

**Examples:** Use properties of logarithms to find the exact value of each expression. (Do not use a calculator).

a)  $\log_6 18 - \log_6 3$

b)  $5^{\log_5 6 + \log_5 7}$

c)  $e^{\log_{e^2} 9}$

d)  $\log_3 8 \cdot \log_8 9$

**Examples:** Write each expression as a sum/difference of logarithms. Express powers as factors.

a)  $\log_7 (x^5)$

b)  $\ln(xe^x)$

$$\text{c) } \log_2 \left( \frac{a}{b^2 c} \right), \quad a > 0, \quad b > 0, \quad c > 0$$

$$\text{d) } \ln \left[ \frac{(x-4)^2}{x^2-1} \right]^{2/3}; \quad x > 4$$

**Examples:** Write each expression as a single logarithm.

$$\text{a) } 3\log_5 u + 4\log_5 v$$

$$\text{b) } \log_4 (x^2 - 1) - 5\log_4 (x + 1)$$

$$\text{c) } \log \left( \frac{x^2 - 2x - 3}{x^2 - 4} \right) - \log \left( \frac{x^2 + 7x + 6}{x + 2} \right)$$

$$\text{d) } 21\log_3 \sqrt[3]{x} + \log_3 (9x^2) - \log_3 9$$

$$\text{e) } \frac{1}{3}\log(x^3 + 1) + \frac{1}{2}\log(x^2 + 1)$$